9.8 Definite Integral

Definite integral of a function: $\int_a^b f(x)dx$, $\int_a^b g(x)dx$, ...

Riemann sum: $\sum_{i=1}^{n} f(\xi_i)\Delta x_i$

Small changes: Δx_i

Antiderivatives: $F(x)$, $G(x)$

Limits of integrations: a, b, c, d

1053. $\int_a^b f(x)dx = \lim_{\substack{n\to\infty \\ \max \Delta x_i \to 0}} \sum_{i=1}^{n} f(\xi_i)\Delta x_i$,

where $\Delta x_i = x_i - x_{i-1}$, $x_{i-1} \le \xi_i \le x_i$.

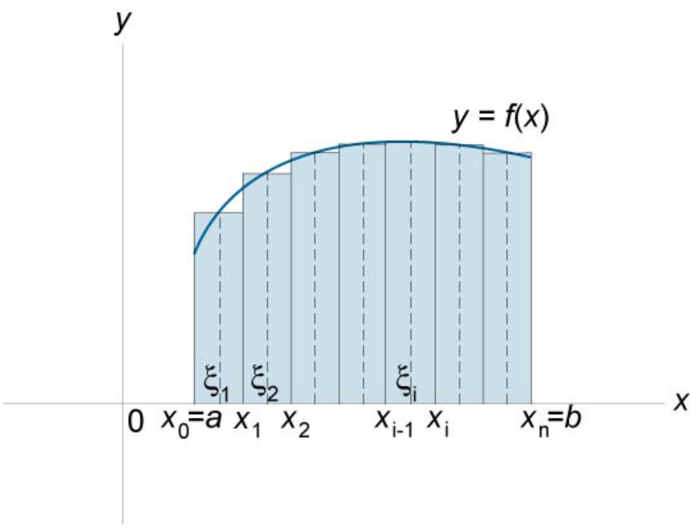

Figure 179.

1054. $\int_a^b 1dx = b - a$

1055. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$

1056. $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

1057. $\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

1058. $\int_a^a f(x)dx = 0$

1059. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

1060. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ for $a < c < b$.

1061. $\int_a^b f(x)dx \geq 0$ if $f(x) \geq 0$ on $[a,b]$.

1062. $\int_a^b f(x)dx \leq 0$ if $f(x) \leq 0$ on $[a,b]$.

1063. Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \text{ if } F'(x) = f(x).$$

1064. Method of Substitution

If $x = g(t)$, then

$$\int_a^b f(x)dx = \int_c^d f(g(t))g'(t)dt,$$

where

$c = g^{-1}(a),\ d = g^{-1}(b).$

1065. Integration by Parts

$$\int_a^b udv = (uv)\Big|_a^b - \int_a^b vdu$$

1066. Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{b-a}{2n}\left[f(x_0) + f(x_n) + 2\sum_{i=1}^{n-1} f(x_i)\right]$$

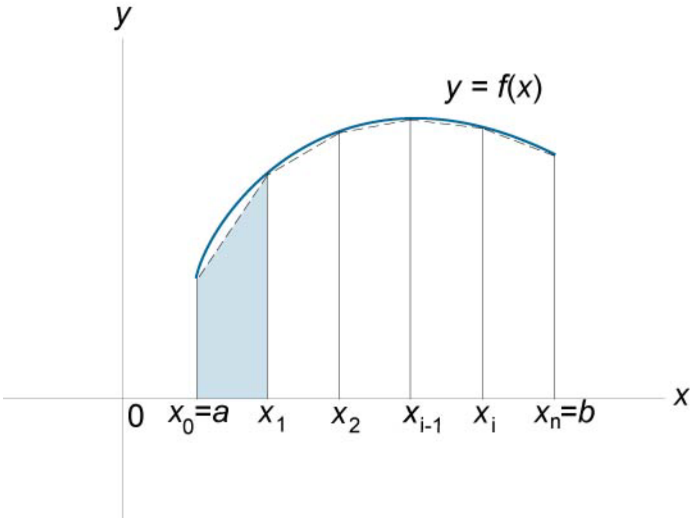

Figure 180.

1067. Simpson's Rule

$$\int_a^b f(x)dx = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \right.$$

$$\left. + 2f(x_4) + \ldots + 4f(x_{n-1}) + f(x_n)\right],$$

where

$$x_i = a + \frac{b-a}{n} i, \; i = 0, 1, 2, \ldots, n.$$

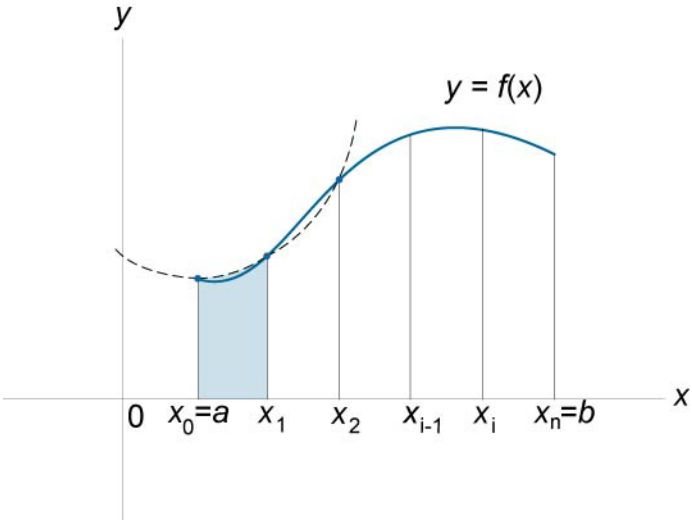

Figure 181.

1068. Area Under a Curve

$$S = \int_a^b f(x)dx = F(b) - F(a),$$

where $F'(x) = f(x)$.

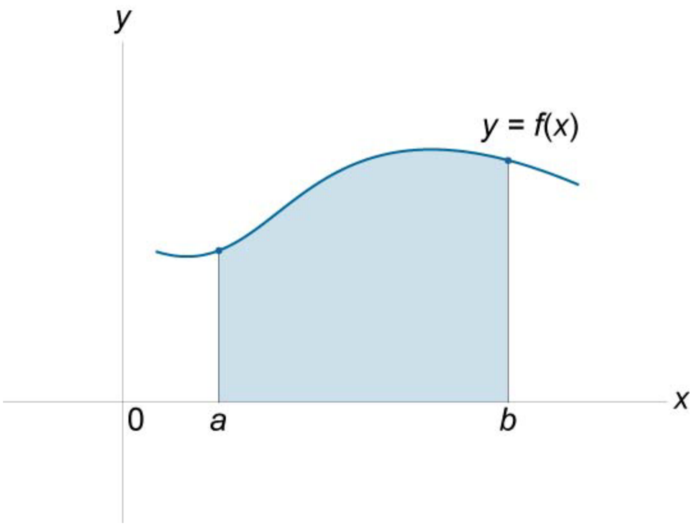

Figure 182.

1069. Area Between Two Curves

$$S = \int_a^b [f(x) - g(x)]dx = F(b) - G(b) - F(a) + G(a),$$

where $F'(x) = f(x)$, $G'(x) = g(x)$.

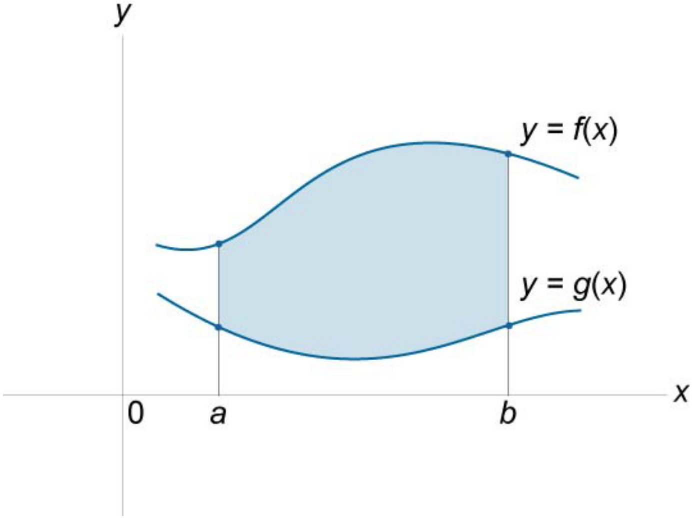

Figure 183.